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Six-loop renormalization group functions of $O(n)$-symmetric ϕ^6-theory and ϵ-expansions of tricritical exponents up to ϵ^3

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Abstract
We present a six-loop calculation for Wilson's β-function and for the crossover exponent φ of the tricritical $O(n)$ symmetric ϕ^6 theory in $d = 3 - \epsilon$ dimensions. The counterterms of all but one of the 29 diagrams can be calculated analytically, while for one diagram high-precision numerical calculations together with the PSLQ integer relation search algorithm was used to come up with an analytical result. The divergences of the dimensionally regularized diagrams are removed by minimal subtraction.

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Renormalized Euclidean quantum field theory has been used to understand critical phenomena during the last 30 years with remarkable success [1–4]. The renormalization group elucidated the importance of dilatation invariance at a critical point and provided a theoretical basis for scaling laws and critical point universality. Multiloop calculations of critical properties in the framework of scalar $O(n)$-symmetric ϕ^4-theory [4, 5] are in excellent quantitative agreement with the most precise measurements on the λ-transition in ^{4}He [6]. In systems with more than one order parameter, lines of critical points can intersect in a multicritical point, giving rise to more complicated scaling laws which are still not completely understood. Focusing on tricritical behaviour, which can be described by a ϕ^4–ϕ^6-theory [7], the renormalization group predicts mean field behaviour at the tricritical point, with universal logarithmic corrections to scaling in the tricritical region [7–11]. Due to the slow crossover to the Gaussian fixed point, it proved to be a rather difficult task to check the asymptotic RG predictions of ϕ^6-theory close to the tricritical point [11] in experiments and simulations. The calculation of the RG flow functions at the six-loop level, which we present here, provides the input for a theoretical description that describes the tricritical region with improved accuracy. In recent years our understanding of the mathematical structures underlying perturbative quantum field theory has made considerable progress by mapping Feynman diagrams of field theories with even upper critical dimension (such as ϕ^4 or ϕ^3) to positive prime knots, thereby connecting the types of knots found in a diagram to the transcendental numbers in the diagram's counterterm (see [12] and references therein). For field theories with odd upper critical dimension, such as ϕ^6, different transcendental numbers show up in the counterterms, and understanding their connection to knots may provide a deeper understanding of the as yet empirical connection between Feynman diagrams, knots and numbers. The calculation of the counterterms for the six-loop diagrams we present here provides a database to undertake this task.

We investigate an $O(n)$-symmetric theory of n real scalar fields ϕ^a ($a \in \{1, \ldots, n\}$) with the Lagrangian

$$L = \frac{1}{2}\partial_k\phi^a\partial_k\phi^a + \frac{m_0^2}{2}\phi^a\phi^a + \frac{u_0}{4!}(\phi^a\phi^a)^2 + \frac{w_0}{6!}(\phi^a\phi^a)^3 \tag{1}$$

in a Euclidean space with $d = 3 - \epsilon$ dimensions. Power counting reveals that both mass and four-point coupling are relevant perturbations at $d = 3$, while the six-point coupling is a marginal (irrelevant) perturbation. We perform a diagrammatic expansion in the six-point coupling and use it to calculate the (tricritical) renormalization of propagator, mass and four-point coupling perturbational. The bare field ϕ, mass m_0 and couplings u_0 and w_0 are expressed via renormalized variables by

$$\phi = Z_\phi^{1/2} l_R^{1-d/2}\phi_R \tag{2}$$

$$m_0^2 - m_{0c}^2 = Z_{m^2} l_R^{-2} m_R^2 = \frac{Z_2}{Z_\phi} l_R^{-2} m_R^2 \tag{3}$$

$$u_0 - u_{0c} = Z_u l_R^{(d-4)} u_R = \frac{Z_4}{Z_\phi^2} l_R^{(d-4)} u_R \tag{4}$$

$$\frac{w_0}{2^5\pi^2} = Z_w l_R^{2(d-3)} \bar{w}_R = \frac{Z_6}{Z_\phi^3} l_R^{2(d-3)} \bar{w}_R. \tag{5}$$

Here l_R is an arbitrary length scale introduced to make all renormalized quantities dimensionless, and a geometric factor $1/(2^5\pi^2)$ has been absorbed in the renormalized coupling $\bar{w}_R$. The Z-factors Z_6, Z_4, Z_2 and Z_ϕ are defined in order to remove the divergences, which occur in the limit $\epsilon \to 0$, from the vertex functions $\Gamma^{(L,N,M)}$ with L insertions of ϕ^2, N insertions of ϕ^4 and M external legs. Applying Bogoliubov's incomplete $\bar{R}$-operation [13, 14], which recursively subtracts the divergences of all subdiagrams, and the $\mathcal{K}$ operation, which selects the pole part of the Laurent series expansion in ϵ of an expression, we use

$$Z_\phi = 1 - \frac{\partial}{\partial k^2}\mathcal{K}\bar{R}\Gamma^{(0,0,2)}\left(\boldsymbol{k}, m_R^2, w_R\right) \tag{6}$$

$$Z_2 = 1 - \frac{\partial}{\partial m_R^2}\mathcal{K}\bar{R}\Gamma^{(0,0,2)}\left(\boldsymbol{k}, m_R^2, w_R\right) \tag{7}$$

$$Z_4 = 1 - \frac{1}{u_R}\mathcal{K}\bar{R}\Gamma^{(0,1,4)}\left(\boldsymbol{k}_i, m_R^2, w_R\right) \tag{8}$$

$$Z_6 = 1 - \frac{1}{w_R}\mathcal{K}\bar{R}\Gamma^{(0,0,6)}\left(\boldsymbol{k}_i, m_R^2, w_R\right) \tag{9}$$

for the calculation of the Z-factors of interest. The equations (2)–(5) together with (6)–(9) then define the renormalized theory, which is finite in the limit $\epsilon \to 0$. This renormalization scheme of minimal subtraction of ϵ-poles for dimensionally regularized vertex functions is described in great detail in [4, 13, 14] and was explained in the present context of ϕ^6-theory in [11]. Now Z_ϕ and Z_2 have already been calculated to six-loop order in [11], so we focus on Z_4 and Z_6 here. At six-loop level the 29 diagrams drawn in figure 1 potentially contribute

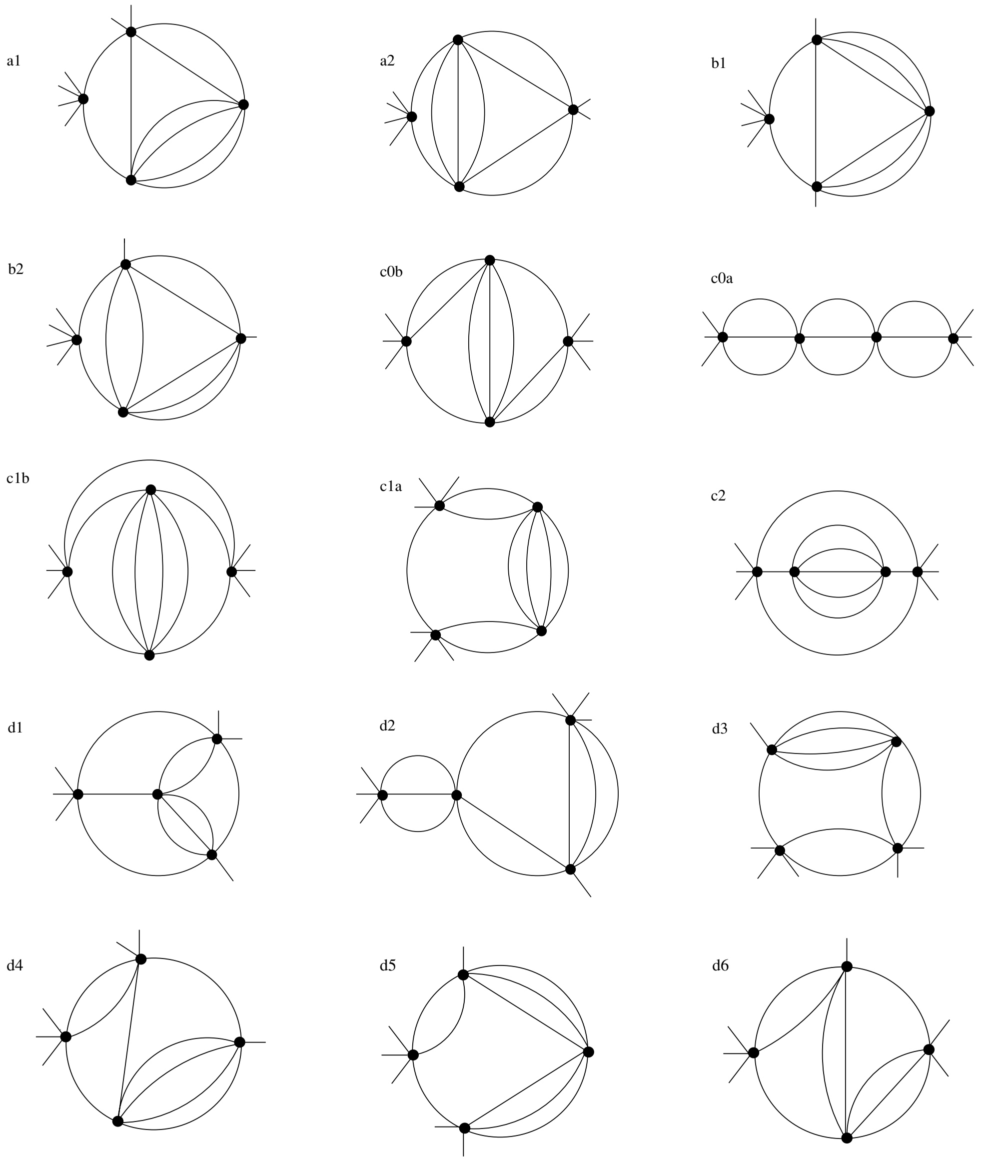

Figure 1. Twenty-nine six-loop diagrams contributing to the vertex function $\Gamma^{(0,0,6)}$.

to the divergences of the vertex function $\Gamma^{(0,0,6)}$ and therefore have to be considered in the calculation of Z_6. By infrared rearrangement [15], using one external momentum $\boldsymbol{k}$ for regularization, we are able to make primitive all but six diagrams, which are of tetrahedron topology. The primitive diagrams are easily calculated in terms of G-functions [14], although some of them required the use of the $\bar{R}^*$-operation [16] to subtract artificial IR-divergences introduced by the IR-rearrangement. Five of the six diagrams of tetrahedron topology were calculated via integration by parts [17] or with a new recursion relation [18]. The remaining diagram (g2b) was reduced to a double sum via expansion in Gegenbauer polynomials [19] and then calculated to a precision of 300 significant figures using recursion relations [20] and convergence acceleration methods [21]. The numerical result for the double sum was

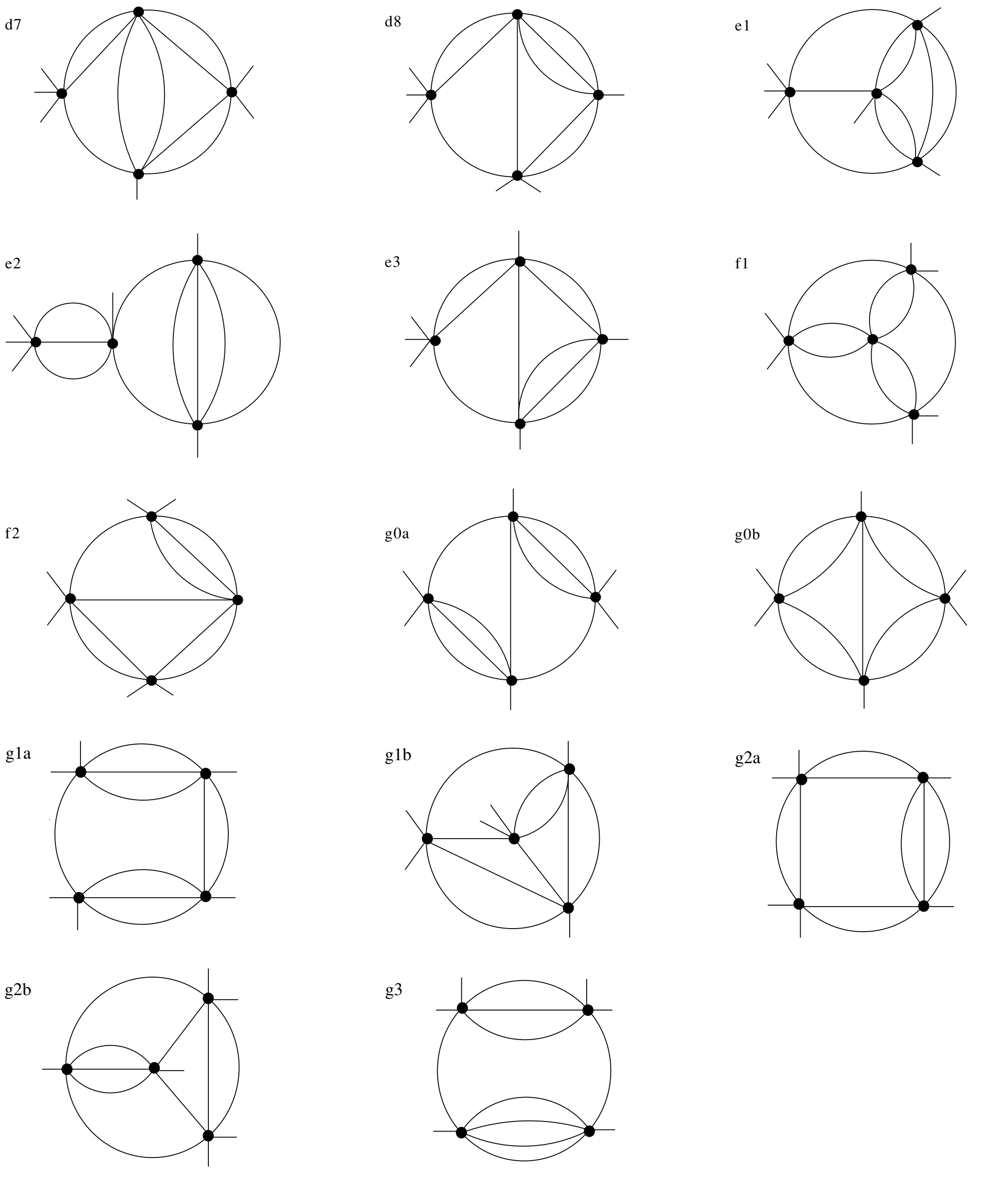

Figure 1. (Continued.)

then matched to the analytical expression $\frac{4}{3}[24\beta(4) + \pi^2 G]$ at 20 significant figures using the integer relation search algorithm PSLQ [22]. Here $\beta(x)$ denotes Dirichlet's beta function $\beta(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^x}$ and $G = \beta(2)$ is Catalan's constant. The perfect match of the remaining 280 digits gives strong evidence that this is indeed a correct result.

The symmetry factor $S = S_1 G_n$ of each diagram can be split into an n-dependent group factor, G_n, from the summation over internal spin indices and a combinatorial factor, S_1, which was calculated from the formula [23]

$$S_1 = \frac{1}{2!^{S+D} 3!^T 4!^F 5!^V N_P} \frac{6!}{\sum_{i=1}^{m} n_i!} \tag{10}$$

where m is the number of vertices, S and D are the number of self- and double connections between vertices, T, F and V are the numbers of triple, fourfold and fivefold connections, respectively, n_i is the number of external legs of vertex i and N_P counts the number of (identical) vertex permutations, that leave the diagram unchanged. G_n was calculated by summing the internal spin indices with an $O(n)$-symmetric interaction for each diagram on a computer.

To obtain the diagrams contributing to $\Gamma^{(0,1,4)}$ at the six-loop level, we have to remove two external vertices from a ϕ^6-vertex from each diagram of figure 1 in all different possible ways. This gives 41 diagrams with one ϕ^4-vertex and four external legs. Since the counterterms are unchanged by this operation, only the combinatorial factors need to be calculated via (10), with 6! changed to 4!, and a proper replacement of one ϕ^6-vertex by a ϕ^4-vertex in the summation over spin indices. Insertion of our results together with the four-loop calculation of [11] into (8) and (9) gives

$$
\begin{aligned}
Z_4 = 1 &+ \frac{1}{\epsilon}\frac{2(n+4)}{15}\bar{w} + (n+4)\left[\frac{1}{\epsilon^2}\frac{(n+6)}{45} - \frac{1}{\epsilon}\left(\frac{\pi^2(n^2+18n+116)}{7200} + \frac{(19n+126)}{600}\right)\right]\bar{w}_R^2 \\
&+ (n+4)\left[\frac{1}{\epsilon^3}\frac{4(n+6)(2n+13)}{2025} - \frac{1}{\epsilon^2}\left(\frac{277n^2+3819n+13\,034}{20\,250}\right.\right. \\
&+ \left.\frac{\pi^2}{81\,000}(4n^3+105n^2+1274n+4692)\right) + \frac{1}{\epsilon}\left(\frac{1}{30\,375}(686n^2+10\,425n\right. \\
&+ 38\,914) + \frac{1}{243\,000}\pi^2(45n^3+1258n^2+13\,168n+43\,204) \\
&- \frac{1}{81\,000}\pi^2\ln(2)(3n^3+19n^2-564n-3508) \\
&+ \frac{1}{162\,000}\pi^4(n^3+40n^2+440n+1544) - \frac{7}{2250}\zeta(3)(n+14)(2n+13) \\
&+ \left.\left.\frac{1}{10\,125}(24\beta(4)+\pi^2G)(7n^2+132n+536)\right)\right]\bar{w}_R^3 + \mathcal{O}\big(\bar{w}_R^4\big)
\end{aligned}
\qquad (11)
$$

$$
\begin{aligned}
Z_6 = 1 &+ \frac{1}{\epsilon}\frac{(3n+22)}{15}\bar{w}_R + \left[\frac{1}{\epsilon^2}\frac{9n^2+132n+484}{225}\right. \\
&- \left.\frac{1}{\epsilon}\left(\frac{\pi^2(n^3+34n^2+620n+2720)}{7200} + \frac{71n^2+1146n+4408}{1200}\right)\right]\bar{w}_R^2 \\
&+ \left[\frac{1}{\epsilon^3}\frac{(3n+22)^3}{3375} - \frac{1}{\epsilon^2}\left(\frac{(3n+22)}{162\,000}(1489n^2+24\,054n+92\,552)\right.\right. \\
&+ \left.\frac{7\pi^2}{324\,000}(3n+22)(n^3+34n^2+620n+2720)\right) \\
&+ \frac{1}{\epsilon}\left(\frac{1}{60\,750}(2787n^3+68\,984n^2+551\,652n+1425\,952)\right. \\
&+ \frac{1}{162\,000}\pi^2(36n^4+1607n^3+33\,568n^2+273\,772n+735\,392) \\
&- \frac{1}{27\,000}\pi^2\ln(2)(n^4+n^3-700n^2-8236n-24\,816) \\
&+ \frac{1}{129\,600}\pi^4(n^4+64n^3+1352n^2+12\,248n+36\,960)
\end{aligned}
$$

$$- \frac{7}{5400}\zeta(3)(11n^3 + 428n^2 + 4228n + 12\,208) + \frac{1}{20\,250}(24\beta(4) + \pi^2 G)$$
$$\times (31n^3 + 1126n^2 + 11\,876n + 37\,592)\Big)\Big]\,\bar{w}_R^3 + \mathcal{O}\big(\bar{w}_R^4\big). \qquad (12)$$

The β function and the exponent functions η, ν and φ are then obtained from the standard definitions

$$\beta(\bar{w}_R) = -l_R \frac{\partial \bar{w}_R}{\partial l_R} = \frac{-2\epsilon \bar{w}_R}{1 + \bar{w}_R \frac{\partial \ln Z_w}{\partial \bar{w}_R}} \qquad (13)$$

$$\eta(\bar{w}_R) = -l_R \frac{\partial}{\partial l_R} \ln Z_\phi = \beta(\bar{w}_R) \frac{\partial \ln Z_\phi}{\partial \bar{w}_R} \qquad (14)$$

$$\gamma_{m^2}(\bar{w}_R) = l_R \frac{\partial}{\partial l_R} \ln Z_{m^2} = -\beta(\bar{w}_R) \frac{\partial \ln Z_{m^2}}{\partial \bar{w}_R} \qquad (15)$$

$$\gamma_u(\bar{w}_R) = l_R \frac{\partial}{\partial l_R} \ln Z_u = -\beta(\bar{w}_R) \frac{\partial \ln Z_u}{\partial \bar{w}_R} \qquad (16)$$

together with the relations [7]

$$\gamma_{m^2} = 2 - \frac{1}{\nu} \qquad \varphi = \nu(1 + \epsilon - \gamma_u). \qquad (17)$$

Using (2), (12) and the result for Z_ϕ given in [11], we find for the Wilson function (13)

$$\beta(\bar{w}_R) = -2\epsilon \bar{w}_R + \frac{6n + 44}{15}\bar{w}_R^2 - \left(\frac{\pi^2(n^3 + 34n^2 + 620n + 2720)}{1800} + \frac{53n^2 + 858n + 3304}{225}\right)$$
$$\times \bar{w}_R^3 + \Bigg(\frac{1}{6750}(1857n^3 + 45\,976n^2 + 367\,716n + 950\,576)$$
$$+ \frac{1}{27\,000}\pi^2(36n^4 + 1607n^3 + 33\,568n^2 + 273\,772n + 735\,392)$$
$$- \frac{1}{4500}\pi^2 \ln(2)(n^4 + n^3 - 700n^2 - 8236n - 24\,816)$$
$$+ \frac{1}{21\,600}\pi^4(n^4 + 64n^3 + 1352n^2 + 12\,248n + 36\,960)$$
$$- \frac{7}{900}\zeta(3)(11n^3 + 428n^2 + 4228n + 12\,208) + \frac{1}{3375}(24\beta(4)$$
$$+ \pi^2 G)(31n^3 + 1126n^2 + 11\,876n + 37\,592)\Bigg)\,\bar{w}_R^4 + \mathcal{O}\big(\bar{w}_R^5\big). \qquad (18)$$

Similarly we find for the exponent functions

$$\eta(\bar{w}_R) = \frac{(n+2)(n+4)}{2700}\bar{w}_R^2 - \frac{(n+2)(n+4)(3n+22)}{60\,750}\bar{w}_R^3 + \mathcal{O}\big(\bar{w}_R^4\big) \qquad (19)$$

$$\nu(\bar{w}_R) = \frac{1}{2} + \frac{(n+2)(n+4)}{675}\bar{w}_R^2 - \left(\frac{\pi^2(n^4 + 24n^3 + 172n^2 + 480n + 448)}{216\,000}\right.$$
$$\left. + \frac{3n^3 + 40n^2 + 156n + 176}{2430}\right)\bar{w}_R^3 + \mathcal{O}\big(\bar{w}_R^4\big) \qquad (20)$$

$$\gamma_u(\bar{w}_R) = \frac{4(n+4)}{15}\bar{w}_R - \frac{(n+4)}{5400}(3\pi^2(n^2+18n+116)+8(85n+566))\bar{w}_R^2 + (n+4)\Bigg(\frac{1}{30\,375}(4113n^2+62\,522n+233\,440) + \frac{1}{40\,500}\pi^2(45n^3+1258n^2+13\,168n+43\,204) - \frac{1}{13\,500}\pi^2\ln(2)(3n^3+19n^2-564n-3805) + \frac{1}{27\,000}\pi^4(n^3+40n^2+440n+1544) - \frac{7}{375}\zeta(3)(n+14)(2n+13) + \frac{2}{3375}(24\beta(4)+\pi^2 G)(7n^2+132n+536)\Bigg)\bar{w}_R^3 + \mathcal{O}\big(\bar{w}_R^4\big). \tag{21}$$

In the ϵ-expansion the nontrivial infrared-stable fixed point emerging for $d < 3$ is given by

$$\bar{w}_R^* = \frac{15\epsilon}{3n+22} + \frac{15}{16}\frac{\pi^2(n^3+34n^2+620n+2720)+424n^2+6864n+26\,432}{(3n+22)^3}\epsilon^2 + \frac{1}{(3n+22)^5}\Bigg(\frac{15}{4}(47n^4+3114n^3+58\,156n^2+397\,848n+920\,160) - \frac{15}{16}\pi^2(293n^4+5386n^3-17\,100n^2-535\,320n-1795\,136) - \frac{45}{8}\pi^2\ln(2)(3n+22)(n^4+n^3-700n^2-8236n-24\,816) + \frac{15}{64}\pi^4(19n^5+128n^4+3520n^3+47\,760n^2+215\,280n+366\,400) + \frac{1575}{8}\zeta(3)(3n+22)(11n^3+428n^2+4228n+12\,208) - \frac{15}{2}(24\beta(4) + \pi^2 G)(3n+22)(31n^3+1126n^2+11\,876n+37\,592)\Bigg)\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{22}$$

The fixed point values of the exponents take the form

$$\eta(\bar{w}_R^*) = \frac{1}{12}\frac{(n+2)(n+4)}{(3n+22)^2}\epsilon^2 \times\left(1+\frac{\pi^2(3n^3+102n^2+1860n+8160)+1128n^2+18\,480n+71\,552}{24(3n+22)^2}\epsilon\right) + \mathcal{O}(\epsilon^4) \tag{23}$$

$$\nu(\bar{w}_R^*) = \frac{1}{2}+\frac{1}{3}\frac{(n+2)(n+4)}{(3n+22)^2}\epsilon^2 + \frac{\pi^2(-3n^5+114n^4+10\,572n^3+114\,072n^2+403\,584n+433\,536)}{576(3n+22)^4}\epsilon^3 + \frac{(2976n^4+76\,992n^3+625\,792n^2+1956\,096n+1\,977\,344)}{576(3n+22)^4}\epsilon^3 + \mathcal{O}(\epsilon^4) \tag{24}$$

$$\varphi(\bar{w}_R^*) = \frac{1}{2} - \frac{(n-6)}{2(3n+22)}\epsilon + \frac{(n+4)(\pi^2(n^3+8n^2+496n+2888) - 8(19n^2+508n+2428))}{16(3n+22)^3}\epsilon^2 + \frac{(n+4)}{(3n+22)^5}\Bigg((185n^4+5423n^3+50\,190n^2+171\,156n+140\,696) - \frac{1}{192}\pi^2(267n^5+2784n^4-231\,552n^3-3463\,216n^2-16\,666\,288n - 27\,085\,120) + \frac{1}{8}\pi^2\ln(2)(3n+22)(3n^4+117n^3+2926n^2+26\,484n + 71\,720) + \frac{1}{128}\pi^4(n^6+30n^5+740n^4+19\,416n^3+215\,120n^2+1132\,896n + 2428\,672) - \frac{21}{4}\zeta(3)(3n+22)(19n^3+1138n^2+12\,452n+37\,016) + 2(24\beta(4)+\pi^2 G)(3n+22)(5n^3+288n^2+3682n+12\,900)\Bigg)\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{25}$$

Our ϵ-expansions coincide with earlier results for η and φ to order ϵ^2 [24], obtained in a massive RG-scheme with cut-off regularization.

For $n = 2$, corresponding to ^{3}He–^{4}He mixtures, the series in $\bar{w}_R$ have the numerical form:

$$\beta(\bar{w}_R) = -2\epsilon\bar{w}_R + 3.733\,33\bar{w}_R^2 - 45.756\,03\bar{w}_R^3 + 1604.926\,64\bar{w}_R^4 + \cdots \tag{26}$$

$$\eta = 0.008\,89\bar{w}_R^2 - 0.011\,06\bar{w}_R^3 + \cdots \tag{27}$$

$$\nu = 0.5 + 0.035\,55\bar{w}_R^2 - 0.381\,82\bar{w}_R^3 + \cdots \tag{28}$$

$$\gamma_u = 1.6\bar{w}_R - 11.674\,41\bar{w}_R^2 + 313.393\,35\bar{w}_R^3 + \cdots. \tag{29}$$

The $n = 2$ ϵ-expansions read, numerically,

$$\eta = 0.002\,551\epsilon^2 + 0.031\,798\epsilon^3 + \cdots \tag{30}$$

$$\nu = 0.5 + 0.010\,204\epsilon^2 + 0.075\,292\epsilon^3 + \cdots \tag{31}$$

$$\varphi = 0.5 + 0.071\,428\epsilon - 1.128\,473\epsilon^2 + 13.907\,664\epsilon^3 + \cdots. \tag{32}$$

As a general trend, the coefficients in the $\bar{w}_R$ and ϵ-expansions show already a pronounced growth roughly by a factor 10 in each order. This indicates that the strong asymptotic divergence proportional to $(2k)!$ (compared to $k!$ for a ϕ^4-theory) of the coefficients in kth order perturbation theory, predicted by instanton methods [25, 26], shows up already in our comparatively short series. We can quantify this statement a little bit by comparing the actual ratios $\beta_2/\beta_1 = -12.26$ and $\beta_3/\beta_2 = -35.08$ of the consecutive expansion coefficients in (26) with the asymptotic behaviour for large order k of perturbation theory of a ϕ^{2r}-theory

$$\beta_k = [k(r-1)]!a^k k^b c\left(1 + \mathcal{O}\left(\frac{1}{k}\right)\right) \tag{33}$$

which gives $\beta_2/\beta_1 = -39.13$ and $\beta_3/\beta_2 = -26.80$, with $a = -64/45\pi^2$ and $b = (7+n)/2$ in the case of $n = 2$ [25, 26]. Since the divergence of the expansion coefficients has already the right order of magnitude, one can hope to extract reasonable approximants for the RG-functions by applying Borel resummation or other resummation techniques [27, 28]. We intend to pursue this avenue in applications of our results to dilute polymer solutions in the case $n = 0$ [10, 29], in order to quantitatively describe experiments and simulations of polymer demixing close to the Θ-point.

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